MCT
2007

#184

Sample Examination I

Section I Part A

Directions: Solve each of the following problems, using available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. Calculators may NOT be used on this part of the exam.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If f is a continuous function defined by $f(x) = \begin{cases} x^2 + bx, & x \le 5 \\ 5\sin\left(\frac{\pi}{2}x\right), & x > 5 \end{cases}$ then $b =$

(A) -6 (B) -5 (C) -4 (D) 4 (E) 5

---

2. The graph of $y = 3x^2 - x^3$ has a relative maximum at

(A) $(0,0)$ only (C) $(2,4)$ only (E) $(0,0)$ and $(2,4)$

(B) $(1,2)$ only (D) $(4,-16)$ only

---

3. A line through the origin, rotates around the origin in such a way that the angle, θ, between the line and the positive x-axis changes at the rate of $\frac{d\theta}{dt}$ for time $t \ge 0$. Which expression gives the rate at which the slope of the line is changing?

(A) $\frac{d\theta}{dt}$ (B) $\cos\theta\frac{d\theta}{dt}$ (C) $-\sin\theta\frac{d\theta}{dt}$

(D) $\frac{1}{\cos^2\theta}\frac{d\theta}{dt}$ (E) $\tan\theta\frac{d\theta}{dt}$

---

4. If $f(x) = e^{\sin x}$, how many zeros does $f'(x)$ have on the closed interval $[0, 2\pi]$?

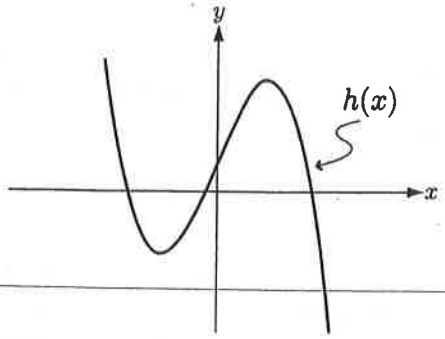
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

5. $\lim_{x\to\infty} \frac{10^8x^5 + 10^6x^4 + 10^4x^2}{10^9x^6 + 10^7x^5 + 10^5x^3} =$

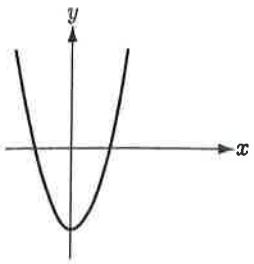
(A) 0 (B) 1 (C) -1 (D) $\frac{1}{10}$ (E) $-\frac{1}{10}$

6.

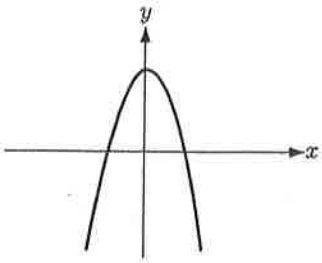


The graph of $h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

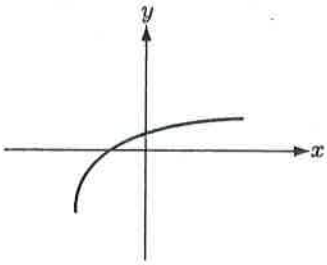
(A)



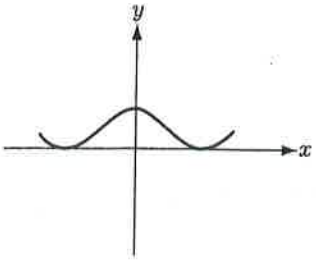
(B)



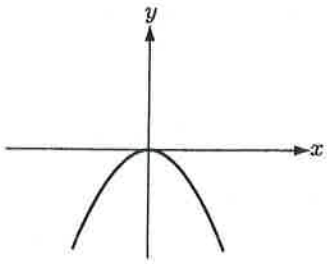
(C)



(D)



(E)



7. If $f(x) = \sqrt{4\sin x + 2}$, then $f'(0) =$

(A) -2 (B) 0 (C) 1 (D) $\frac{\sqrt{2}}{2}$ (E) $\sqrt{2}$

8. If t is measured in hours and $f'(t)$ is measured in knots, then $\int_0^2 f'(t)\,dt =$ (Note: 1 knot = 1 nautical mile/hour)

(A) $f(2)$ knots

(B) $f(2) - f(0)$ knots

~~(C) $f(2)$ nautical miles~~

(D) $f(2) - f(0)$ nautical miles

(E) $f(2) - f(0)$ knots/ hour

9. The equation of the tangent line to the curve $x^2 + y^2 = 169$ at the point $(5, -12)$ is

(A) $5y - 12x = -120$

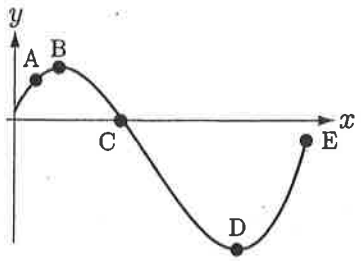
(B) $5x - 12y = 119$

(C) $5x - 12y = 169$

(D) $12x + 5y = 0$

(E) $12x + 5y = 169$

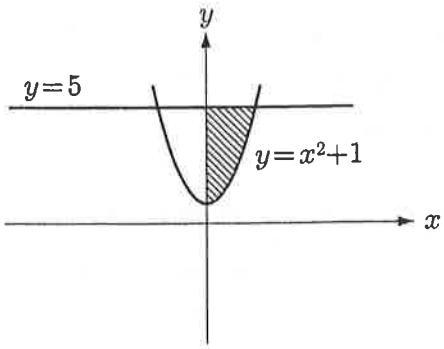
10.



The figure above shows the graph of the velocity of a moving object as a function of time. At which of the marked points is the speed the greatest?

(A) A (B) B (C) C (D) D (E) E

11.



For the figure above, the area of the shaded region is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$

(D) $\frac{32}{3}$ (E) $\frac{65}{3}$

12. $\int \frac{1}{\sqrt{4-x^2}}\,dx =$

(A) $\text{Arcsin}\,\frac{x}{2}+C$ (B) $2\sqrt{4-x^2}+C$ (C) $\text{Arcsin}\,x+C$

(D) $\sqrt{4-x^2}+C$ (E) $\frac{1}{2}\text{Arcsin}\,\frac{x}{2}+C$

13. If the graph of $f(x)=2x^2+\frac{k}{x}$ has a point of inflection at $x=-1$, then the value of k is

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

14. $\int \sin(3x+4)\,dx =$

(A) $-\frac{1}{3}\cos(3x+4)+C$

(B) $-\cos(3x+4)+C$

(C) $-3\cos(3x+4)+C$

(D) $\cos(3x+4)+C$

(E) $\frac{1}{3}\cos(3x+4)+C$

---

15. What are all values of x for which the graph of $y = \dfrac{2}{4-x}$ is concave downward?

(A) No values of x (B) $x<4$ (C) $x>-4$ (D) $x<-4$ (E) $x>4$

---

16. A particle moves along the x-axis in such a way that its position at time t is given by $x(t) = \dfrac{1-t}{1+t}$. What is the acceleration of the particle at time $t=0$?

(A) -4 (B) -2 (C) $-\frac{3}{5}$ (D) 2 (E) 4

---

17. If, for all real numbers x, $f(x) = g(x)+5$, then on any interval $[a,b]$ the area of the region between the graphs of f and g is

(A) 5

(B) $5a+5b$

(C) $5b-5a$

(D) $5a-5b$

(E) $5ab$

---

18. If $y = x(\ln x)^2$, then $\dfrac{dy}{dx} =$

(A) $3(\ln x)^2$

(B) $(\ln x)(2x+\ln x)$

(C) $(\ln x)(2+\ln x)$

(D) $(\ln x)(2+x\ln x)$

(E) $(\ln x)(1+\ln x)$

---

19. If $\int_0^6 (x^2-2x+2)\,dx$ is approximated by three inscribed rectangles of equal width on the x–axis, then the approximation is

(A) 24

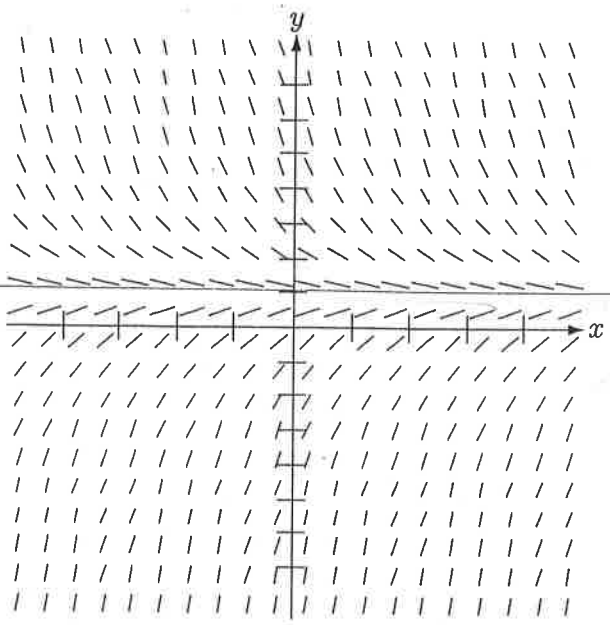
(B) 26

(C) 28

(D) 48

(E) 76

20.



Shown above is the slope field for which differential equation?

(A) $\frac{dy}{dx} = 1 - x$

(B) $\frac{dy}{dx} = x - y$

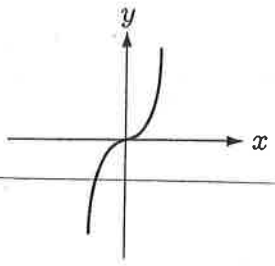
(C) $\frac{dy}{dx} = -\frac{x}{y}$

(D) $\frac{dy}{dx} = \frac{y}{x}$

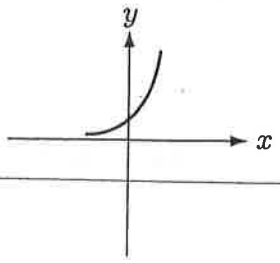
(E) $\frac{dy}{dx} = 1 - y$

21. If, for all real numbers x, $f'(x) < 0$ and $f''(x) > 0$, which of the following curves could be part of the graph f?

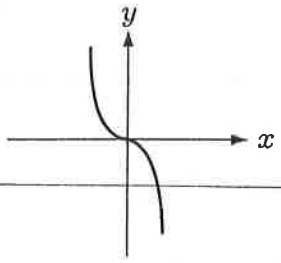
(A)



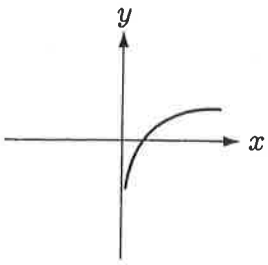
(B)



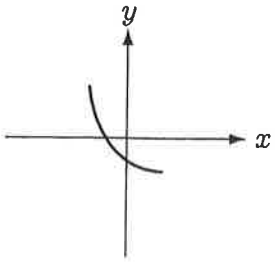
(C)



(D)



(E)



22. If $\frac{dy}{dx} = x^2y^2$, then $\frac{d^2y}{dx^2} =$

(A) $2xy^2$

(B) $4x^3y^3$

(C) $2x + 2x^2y^3$

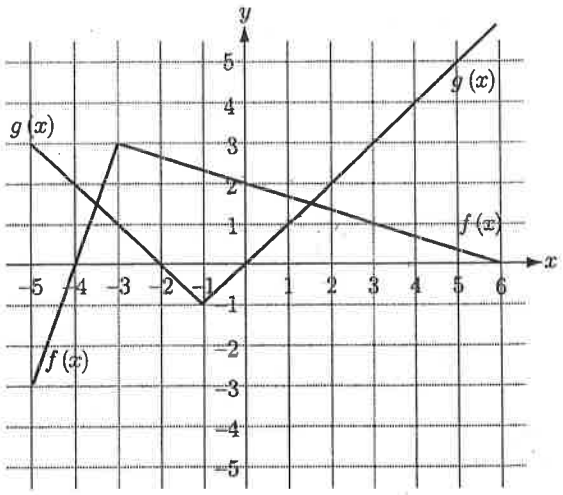
(D) $2x^2y + 2xy^2$

(E) $2x^4y^3 + 2xy^2$

23. $4\int_1^{e^2} \frac{x - x^3}{x^2}\, dx =$

(A) $3 - e^2$ (B) $3 - e^4$ (C) $5 - e^2$ (D) $5 - e^4$ (E) $10 - 2e^4$

24.



The functions f and g are piecewise linear functions whose graphs are shown above. If $h(x) = f(x)g(x)$, then $h'(3) =$

(A) $-\frac{8}{3}$ (B) $-\frac{1}{3}$ (C) 0

(D) $\frac{2}{3}$ (E) $\frac{8}{3}$

25. For which pair of functions $f(x)$ and $g(x)$ below, will the $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$?

	$f(x)$	$g(x)$
(A)	e^x	x^2
(B)	e^x	$\ln x$
(C)	$\ln x$	e^x
(D)	x	$\ln x$
(E)	3^x	2^x

26. Let $f(x)$ be the function defined by $f(x) = \begin{cases} x, & x \le 0 \\ x+1, & x > 0 \end{cases}$.

The value of $\int_{-2}^{1} xf(x)\,dx =$

(A) $\frac{3}{2}$ (B) $\frac{5}{2}$ (C) 3 (D) $\frac{7}{2}$ (E) $\frac{11}{2}$

---

27. The average value of the function $f(x) = \cos\left(\frac{1}{2}x\right)$ on the closed interval $[-4, 0]$ is

(A) $-\frac{1}{2}\sin(2)$ (C) $\frac{1}{2}\cos(2)$ (E) $\frac{1}{2}\sin(2)$

(B) $-\frac{1}{4}\sin(2)$ (D) $\frac{1}{4}\sin(2)$

---

28. If n is a positive integer, then $\lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{1+(1/n)} + \frac{1}{1+(2/n)} + \cdots + \frac{1}{1+(n/n)}\right]$ can be expressed as

(A) $\int_0^1 \frac{1}{x}\,dx$ (B) $\int_1^2 \frac{1}{x+1}\,dx$ (C) $\int_1^2 x\,dx$ (D) $\int_1^2 \frac{2}{x+1}\,dx$ (E) $\int_1^2 \frac{1}{x}\,dx$

Section I Part B

Directions: Solve each of the following problems, using available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. A graphing calculator is required for some questions on this part of the examination.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices, the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. The volume of the solid formed by revolving the region bounded by the graph of $y = (x-3)^2$ and the coordinate axes about the x−axis is given by which of the following integrals?

(A) $\pi\int_0^3 (x-3)^2\,dx$ (C) $2\pi\int_0^3 (x-3)^2\,dx$ (E) $2\pi\int_0^3 x(x-3)^4\,dx$

(B) $\pi\int_0^3 (x-3)^4\,dx$ (D) $2\pi\int_0^3 x(x-3)^2\,dx$

30. Let f be the function given by $f(x) = \tan x$ and let g be the function given by $g(x) = x^2$. At what value of x in the interval $0 \le x \le \pi$ do the graphs of f and g have parallel tangent lines?

(A) 0
(B) 0.660
(C) 2.083
(D) 2.194
(E) 2.207

---

31. Let $f(t) = \frac{1}{t}$ for $t > 0$. For what value of t is $f'(t)$ equal to the average rate of change of f on the closed interval $[a, b]$?

(A) $-\sqrt{ab}$
(B) $\sqrt{ab}$
(C) $-\frac{1}{\sqrt{ab}}$
(D) $\frac{1}{\sqrt{ab}}$
(E) $\sqrt{\frac{1}{2}\left(\frac{1}{b} - \frac{1}{a}\right)}$

32.

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	20	19.5	18	15.5	12	7.5	2	−4.5	−12	−20.5	−30

Selected values of a continuous function are given in the table above. Using 10 subintervals of equal length, the Trapezoidal Rule approximation for $\int_0^{10} f(x)\,dx$ is

(A) 7.500 (B) 32.500 (C) 33.325 (D) 33.333 (E) 57.500

---

33. Let $R(t)$ represent the rate at which water is leaking out of a tank, where t is measured in hours. Which of the following expressions represents the total amount of water in gallons that leaks out in the first three hours?

(A) $R(3) - R(0)$
(B) $\frac{R(3) - R(0)}{3 - 0}$
(C) $\int_0^3 R(t)\,dt$
(D) $\int_0^3 R'(t)\,dt$
(E) $\frac{1}{3}\int_0^3 R(t)\,dt$

---

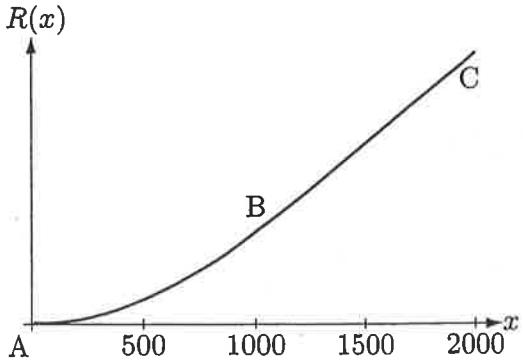
34. Let f and g be differentiable functions such that

$$f(1) = 4, g(1) = 3, f'(3) = -5$$
$$f'(1) = -4, g'(1) = -3, g'(3) = 2$$

If $h(x) = f\big(g(x)\big)$, then $h'(1) =$

(A) −9
(B) 15
(C) 0
(D) −5
(E) −12

35.



The figure above shows a road running in the shape of a parabola from the bottom of a hill at A to point B. At B it changes to a line and continues on to C. The equation of the road is

$$R(x) = \begin{cases} ax^2, & \text{from A to B} \\ bx + c, & \text{from B to C} \end{cases}$$

B is 1000 feet horizontally from A and 100 feet higher. Since the road is smooth, $R'(x)$ is continuous. What is the value of b?

(A) 0.2 (B) 0.02 (C) 0.002 (D) 0.0002 (E) 0.00002

---

36. The area of the region enclosed by the graphs of $y = e^{(x^2)} - 2$ and $y = \sqrt{4 - x^2}$ is

(A) 2.525 (B) 4.049 (C) 4.328 (D) 5.050 (E) 6.289

37.

t(sec)	0	2	4	6	8
$a(t)$ (ft/sec^2)	2	3	4	3	2

The table for the acceleration of a particle from 0 to 8 seconds is given in the table above. If the velocity at $t = 0$ is 4 feet per second, the approximation value of the velocity, in feet per second, at $t = 8$ seconds, computed using the Riemann Sum with four subdivisions of equal length is

(A) 4
(B) 12
(C) 16
(D) 24
(E) 28

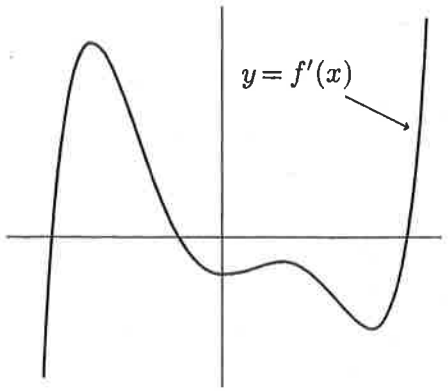
---

38. Suppose that $f(x)$ is an even function and let $\int_0^1 f(x)\,dx = 5$ and $\int_0^7 f(x)\,dx = 1$.

What is $\int_{-7}^{-1} f(x)\,dx$?

(A) -5
(B) -4
(C) 0
(D) 4
(E) 5

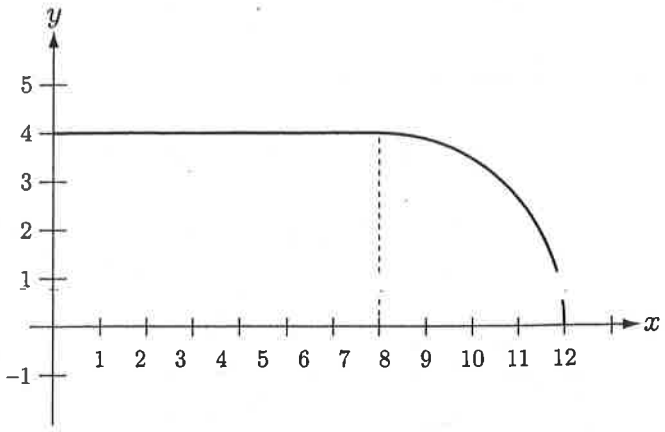
39.



The figure above shows the graph of the derivative of a function f. How many points of inflection does f have in the interval shown?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

40.



As shown in the figure above the function $f(x)$ consists of a line segment from $(0, 4)$ to $(8, 4)$ and one-quarter of a circle with a radius of 4. What is the average (mean) value of this function on the interval $[0, 12]$?

(A) 2
(B) 3.714
(C) 3.9
(D) 22.283
(E) 41.144

41. If f is the function defined by $f(x) = \sqrt[3]{x^2 + 4x}$ and g is an antiderivative of f such that $g(5) = 7$, then $g(1) \approx$

(A) -3.882
(B) -3.557
(C) 1.710
(D) 3.557
(E) 3.882

42. The amount $A(t)$ of a certain item produced in a factory is given by

$$A(t) = 4000 + 48(t-3) - 4(t-3)^3$$

where t is the number of hours of production since the beginning of the workday at 8:00 am. At what time is the rate of production increasing most rapidly?

(A) 8:00 am (B) 10:00 am (C) 11:00 am

(D) 12:00 noon (E) 1:00 pm

43. At how many points on the curve $y = 4x^5 - 3x^4 + 15x^2 + 6$ will the line tangent to the curve pass through the origin?

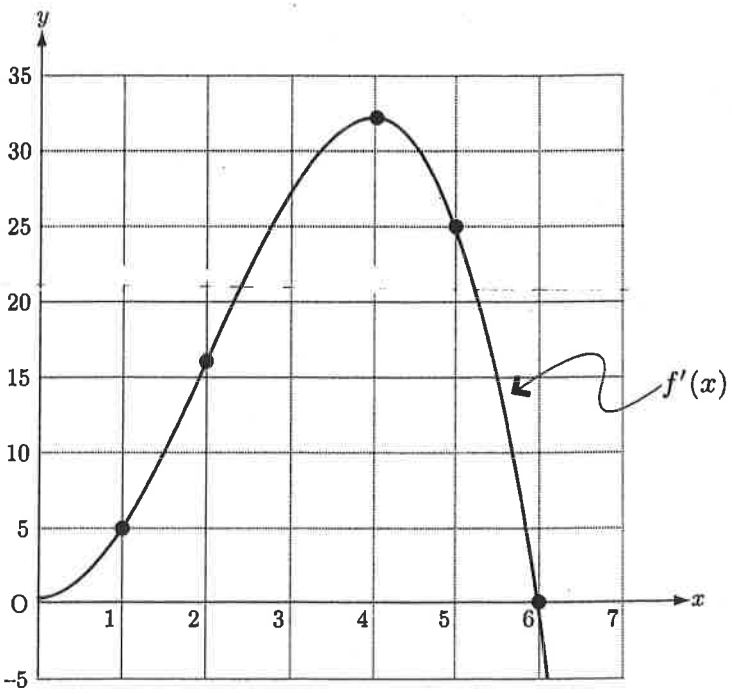
(A) One (B) Two (C) Three

(D) Four (E) Five

44. A population grows according to the equation $P(t) = 6000 - 5500e^{-0.159t}$ for $t \geq 0$, t measured in years. This population will approach a limiting value as time goes on. During which year will the population reach half of this limiting value?

(A) Second (B) Third (C) Fourth (D) Eighth (E) Twenty-ninth

45.



Note: This is the graph of $f'(x)$, NOT the graph of $f(x)$.

Let f be a differentiable function for all x. The graph of $f'(x)$ is shown above. If $f(2) = 10$, which of the following best approximates the maximum value of $f(x)$?

(A) 30 (C) 70 (E) 110

(B) 50 (D) 90